Student Name: ____________________

Student Number: ____________________

Total Marks: ________
100

Okanagan University College
Final Examination

Math 112 (Fall, 2001)

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Section(s): **01, 02, 51, 61, 71, & 72**

001214

1:00PM

Duration: **3 hours**

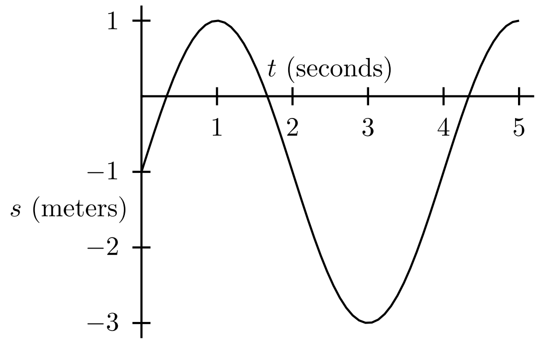
READ INSTRUCTIONS CAREFULLY BEFORE COMMENCING EXAM

INSTRUCTIONS: Answer all 13 questions in the spaces provided, showing all significant steps. Partial marks will be awarded for correct work even if the final answer is incorrect. Marks per question are given in the left margin, total 100. Check that your paper contains all 12 pages in addition to the cover page.

This paper contains pages numbered 1 to 12

EXAM BOOKLETS ARE NOT REQUIRED

1 The following graph represents **displacement** s (in meters) as a function of **time** t (in seconds).



[2] (a) Use the graph to **estimate** the **average velocity** in the time interval $1.5 \leq t \leq 3.5$. Show your calculations and give your answer to one figure to the right of the decimal place. The average velocity is represented by the slope of a certain line. Draw this line (as a dashed line) on the graph.

[2] (b) Use the graph to **estimate** the **instantaneous velocity** at time $t = 1.5$. Show your calculations and give your answer to one figure to the right of the decimal place. The instantaneous velocity is also represented by the slope of a certain line. Draw this line (as a solid line) on the graph.

[2] (c) Are there any times from $t = 0$ to $t = 4$ when the **instantaneous velocity** is zero? If so, what times?

[1] (d) There is an inflection point at $t = 2$. What can you say about the instantaneous velocity at $t = 2$?

2 Calculate the following **derivatives**, **simplify only if instructed to do so**:

[3] (a) $h'(3)$, given $h(t) = t^2 \ln(t+2)$.

[3] (b) $f'(x)$, given $f(x) = 1 + x + \arctan\left(\frac{x}{2}\right)$. **Simplify** your answer.

[3] (c) $\frac{dy}{dx}$, given $y = \left[\sqrt{x^3+2} + \cos(2x)\right]^6$.

Problem 2 continued . . .

. . . Problem 2 continued

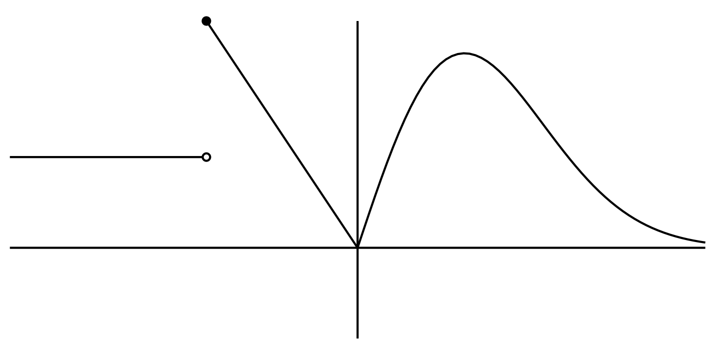
[3] (d) $\dfrac{dI}{dL}$, given $I = \dfrac{V}{\sqrt{R^2 + \omega^2 L^2}}$ where V, R and ω are constants.

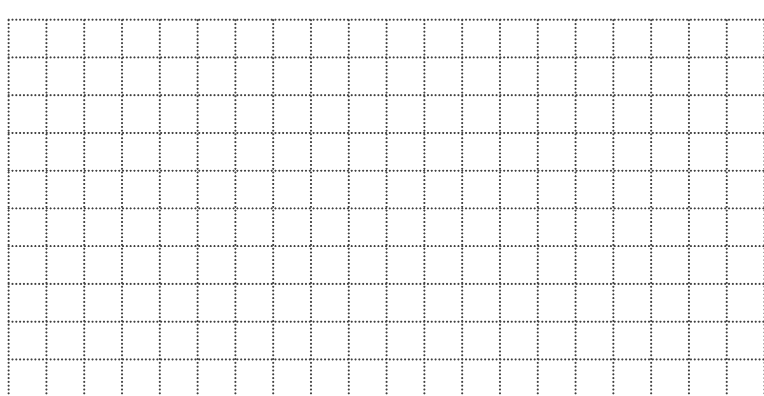
[4] (e) $k''(z)$, given $k(z) = \dfrac{3z-2}{5z+4}$. **Simplify** your answer.

[4] (f) $\dfrac{dy}{dx}$, given $e^{(x^2)} y^3 = 4 + \sin(y)$.

[5] 3 Given $f(x) = \dfrac{x}{x-2}$, use the **limit definition** of the derivative to show that $f'(x) = \dfrac{-2}{(x-2)^2}$.

[4] 4 The following is the graph of some function $f(x)$. **Sketch the graph of the derivative function** $f'(x)$ as accurately as you can on the set of coordinate axes provided.





5 Let $g(x) = x\,2^{h(x)}$, where $h(3) = -2$ and $h'(3) = 5$.

[4] (a) **Find an expression for** $g'(x)$ in terms of $h(x)$ and $h'(x)$.

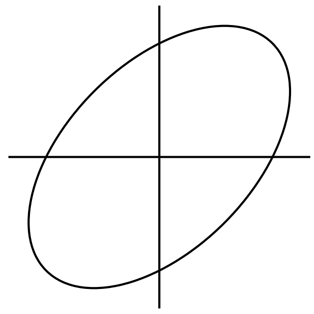
[2] (b) **Give the exact value** of $g'(3)$, not a decimal approximation. Your value will contain $\ln 2$.

[5] 6 Consider the limit

$$\lim_{x \to 3} \frac{x^2 + x + 2a}{x^2 - x - 6}$$

Find the value of a for which this **limit exists** and find the **value of the limit** for this a.

7 The graph of the equation $x^2 - xy + y^2 = 9$ is the "tilted" ellipse shown in the following diagram.



[1] (a) What are the **coordinates of the two points** where the ellipse intersects the x-axis?

[3] (b) Use implicit differentiation to **find** $\frac{dy}{dx}$.

[2] (c) Determine the equations of the **two tangent lines** to the ellipse at the points in part (a).

[1] (d) Do these two tangent lines intersect? If so, find the coordinates of the point of intersection. If not, briefly explain why not.

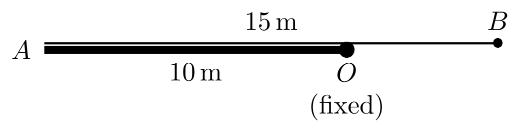
[4] 8 **Verify** that the equation $x + \sin x = 3$ has at least one real solution. Show all work and justify your conclusion.

9 Consider the function $f(x) = \sqrt[5]{2-x}$.

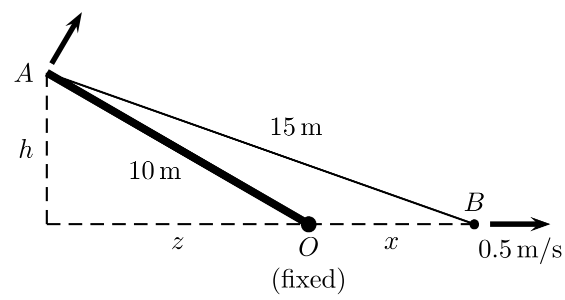
[3] (a) Find the **tangent line approximation** (the **linearization**) of $f(x)$ at $x = 1$.

[2] (b) Use your result in part (a) to **estimate** $\sqrt[5]{1.01}$

10 A 10-metre totem pole will be raised by fixing one end to the ground at point O in the diagram, and attaching a 15-metre rope to the other end, A. The rope is laid out along the totem pole and extended beyond the fixed end at O to the point B.



The end at B is then pulled along the ground at 0.5 m/s directly away from the fixed end of the totem pole raising the end A off the ground.



[3] (a) Use Pythagoras theorem for each of the two right triangles in the second diagram to **express** z **in terms of** x.

[2] (b) Give **the values of** x **and** z when the totem **pole is on the ground** and **ten seconds after the rope starts moving**.

[4] (c) Assuming that the sun is directly overhead, find the **rate at which the length of the shadow** cast by the totem pole is changing ten seconds after the rope starts moving.

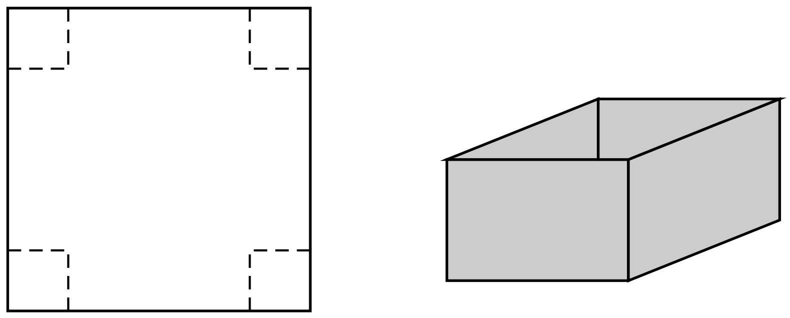
11 **Evaluate each limit**. You may use l'Hopital's rule where appropriate.

[3] (a) $\lim_{x \to 0} \dfrac{e^x + e^{-x} - 2}{1 - \cos(2x)}$

[3] (b) $\lim_{x \to \infty} \dfrac{1 - \ln x}{1 + e^{-x}}$

[3] (c) $\lim_{x \to 0^+} \sqrt{x} \ln x$

[7] 12 A square sheet of paper is to be made into a container by cutting squares out of each of the corners. The container is to have a square base, vertical sides and an open top with a volume of exactly 2000cm^3. **Determine the dimensions of the container** so that the total area of the paper, including the discarded square corners, is minimum. See the diagram. Hint: Can you just minimize the length of one side of the original square piece of paper?



13 A certain function f has domain $(-\infty,-1)\cup(-1,\infty)$ and is continuous on its domain. Further,

$$f(-3)=8,\ f(2)=3,\ f(3)=4$$

[3] (a) The function f described above satisfies:

$$f'(x)>0 \text{ for } x<-3 \text{ and } x>2$$
$$f'(x)<0 \text{ for } -3<x<-1 \text{ and } -1<x<2$$
$$f'(-3) \text{ does not exist}$$

Give the **intervals where f is increasing and decreasing**. Identify all of the **critical numbers** and classify each as a **local maximum, local minimum, or neither**.

[3] (b) The same function f also satisfies:

$$f''(x)>0 \text{ for } x<-3 \text{ and } -1<x<3$$
$$f''(x)<0 \text{ for } -3<x<-1 \text{ and } x>3$$
$$f''(-3) \text{ does not exist}$$

Give the intervals where the graph of f is **concave up and concave down**. Identify all **inflection points**.

Problem 13 continued ...

. . . Problem 13 continued

[3] (c) The same function f satisfies:

$$\lim_{x\to\infty} f(x) = 5 \text{ and } \lim_{x\to-\infty} f(x) = -\infty$$
$$\lim_{x\to-1^-} f(x) = -\infty \text{ and } \lim_{x\to-1^+} f(x) = \infty$$

Using this information together with any relevant information from parts (a) and (b) above determine any **vertical or horizontal asymptotes** of the graph of f. **Describe the behaviour** of the function f near any **vertical asymptotes** and the **end behaviour of the function** for both large positive and large negative x.

[3] (d) **Sketch a possible graph of** f whose properties are given above. Label each **critical number** and **inflection point** with its correct coordinates. Identify any **vertical and/or horizontal asymptotes**.