Student Name: ______________________

Student Number: ______________________

Total Marks: ________
100

Okanagan College
Final Examination

Math 112 Winter 2010

Instructor(s): Clint Lee

Section(s): 071

Thursday, April 22

9:00 AM

Duration: 3 hours

READ INSTRUCTIONS CAREFULLY BEFORE COMMENCING EXAM

INSTRUCTIONS: Answer all 19 questions in the spaces provided, showing all significant steps. Partial marks will be awarded for correct work even if the final answer is incorrect. Marks per question are given in the left margin, total 100. Check that your paper contains all 13 pages in addition to the cover page.

This paper contains pages numbered 1 to 13

EXAM BOOKLETS ARE NOT REQUIRED

1. Differentiate but do not simplify.

[1] (a) $f(x) = \dfrac{1}{\sqrt{x}} + 2e^x$

[1] (b) $g(x) = x^2 \cos x$

[2] 2. Let f be a function for which $f(1) = 2$ and $f'(1) = -3$ and let

$$h(x) = \frac{f(x)}{x+1}$$

Find $h'(1)$.

[3] 3. Use implicit differentiation to find $\dfrac{dy}{dx}$ if $xy^2 + y^3 = x^2$.

[2] 4. (a) A version of the Intermediate Value Theorem is given below with some parts left blank. Fill in each blank with the correct quantity or statement from the selections (A) – (I) given below.

Let f be a function that is ____________ on the interval $[a, b]$. Suppose that $f(a)$ ____________ and $f(b) > 0$. There is a number c in the interval (a, b) for which ____________.

(A) differentiable	(B) continuous	(C) defined
(D) $f(c) = 0$	(E) $f'(c) = 0$	(F) $f\left(\frac{a+b}{2}\right) = c$
(G) < 0	(H) > 0	(I) $= 0$

[2] (b) Use the Intermediate Value Theorem, as stated above, to verify that the equation

$$\ln\left(x^2 + 1\right) = 9 - x^2$$

has at least one solution in the interval $[2, 3]$.

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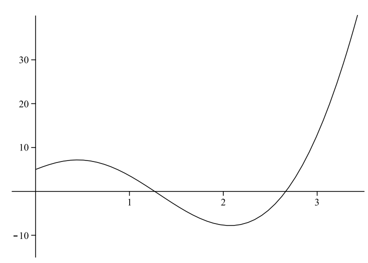
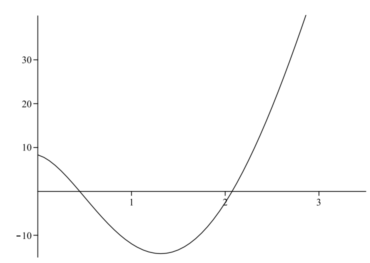
5. The graph of a function f and the graph of its derivative f' are shown below.





[2] (a) Indicate which graph is that of the derivative f'. Explain your choice.

[2] (b) Assuming that the function f gives the position, in metres, of an object moving along a straight line at time t, in seconds, estimate the acceleration of this object at time $t = 2.5\,\mathrm{s}$

6. Evaluate each limit, if it exists. If the limit does not exist, state this and explain why it does not.

[1] (a) $\lim_{x \to 1^-} \left(4x^2 - 3x + 7\right)$

[2] (b) $\lim_{x \to 3} \dfrac{x^2 - 2x - 3}{x^3 - 7x^2 + 12x}$

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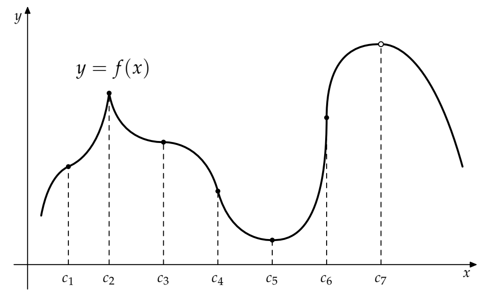
[4] 7. Use the limit definition of the derivative to find the derivative of $f(x) = x + x^2$.

8. The graph of a function f is shown.



[3] (a) Determine which of the x-values, c_1, c_2, c_3, c_4, c_5, c_6, and c_7, are critical numbers of the function f, and classify each critical number as a local maximum, local minimum, or neither.

[2] (b) Determine which of the x-values, c_1, c_2, c_3, c_4, c_5, c_6, and c_7, correspond to an inflection point of the graph of the function f.

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[3] 9. Find the linearization $L(x)$ of the function $f(x) = \sin(\pi x^2)$ valid when $x \approx 1$.

10. A sample of cesium-137 was produced with an initial mass of 80.0 g. After 100 years, the sample had decayed to a mass of 7.94 g.

[2] (a) The mass of the sample after t years is given by $m(t) = Ae^{-kt}$. Determine the values of A and k.

[2] (b) Find the average rate of change of mass with respect to time between 0 and 100 years after the date of production.

[3] (c) Find the instantaneous rate of change of mass with respect to time 50 years after the date of production. Explain in practical terms what this rate of change represents.

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[3] 11. A croquet ball is a sphere with diameter of 8 cm. Use differentials to estimate the volume of paint required to paint with uniform layer of paint 0.1 mm thick. You may use the fact that the volume of a sphere of radius r is $V = \frac{4\pi}{3}r^3$.

12. Consider the function

$$f(x) = \frac{x + e^{-x}}{x - 1}$$

[2] (a) Find all vertical asymptotes of f, using appropriate infinite limits to justify your answer.

[2] (b) Find all horizontal asymptotes of f, using limits at infinity to justify your answer.

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13. Evaluate each limit, if it exists. If the limit does not exist, state this and explain why it does not.

[2] (a) $\displaystyle\lim_{x \to 0} \frac{1 - \cos(x)}{\sin^2(x)}$

[2] (b) $\displaystyle\lim_{x \to \frac{3}{4}} \frac{4x^2 - 3x}{|4x - 3|}$

[2] (c) $\displaystyle\lim_{x \to 5^+} \ln(x^2 - 25)$

[3] (d) $\displaystyle\lim_{t \to 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t}$

14. Find the indicated derivative. Simplify only if instructed to do so.

[3] (a) Find $f'(x)$ for $f(x) = \sqrt{1-x^2}\arcsin x$ and simplify.

[3] (b) Find $z'(t)$ for $z(t) = \sin\left[\ln\left(e^{t^2}+1\right)\right]$.

Problem 14 continued ...

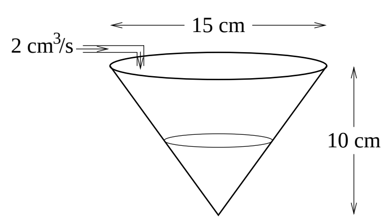
. . . Problem 14 continued

[4] (c) Find $\frac{dy}{dx}$ for $y = \frac{e^{-x}}{e^{-x}+1}$ and simplify.

[3] (d) Find $q'(r)$ for $q(r) = r^2 \sec^2 r$.

[4] 15. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ for function $y = (\sin x)^{\sqrt{x^2+1}}$.

[4] 16. Water is poured at a constant rate of 2 cm^3/s into a cone that has an overall height of 10 cm and a diameter of 15 cm.



Find the rate at which the depth of water in the cone is changing 75 seconds after water begins to pour into the cone. Hint: The volume of a right circular cone with base radius r and height h is $V = \frac{1}{3}\pi r^2 h$.

[3] 17. Determine the values of the constants a and b for which the following function is continuous and differentiable at $x = \dfrac{\pi}{2}$?

$$f(x) = \begin{cases} 2\sin(x) + 1 & \text{if } x \leq \frac{\pi}{2} \\ ax + b & \text{if } x > \frac{\pi}{2} \end{cases}$$

18. Consider the function

$$F(x) = \frac{3\,(x-2)^2}{x^2-9}$$

[4] (a) Verify that the first derivative of this function is:

$$F'\,(x) = \frac{6\,(x-2)\,(2x-9)}{(x^2-9)^2}$$

[4] (b) Find the critical numbers of the function F, and determine the intervals where F is increasing and decreasing. Use the First Derivative Test to classify each critical number as a local maximum, local minimum, or neither.

Problem 18 continued ...

. . . Problem 18 continued

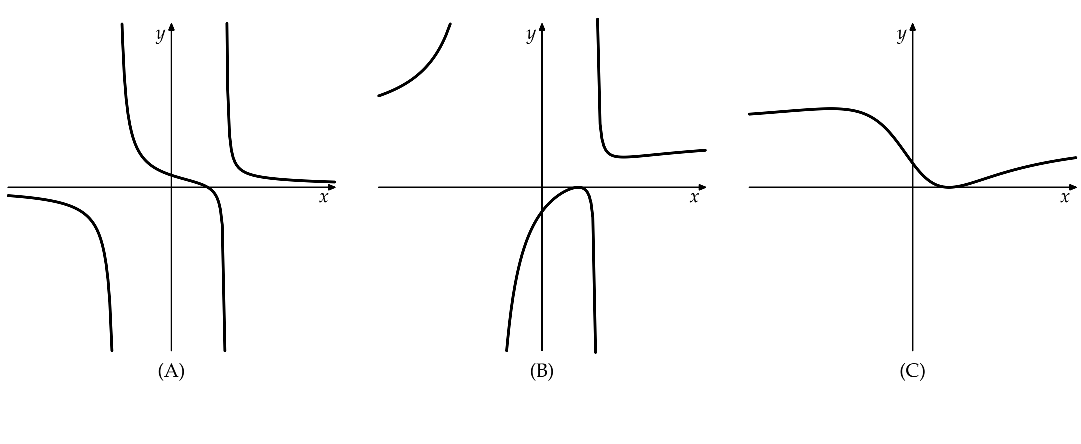
[3] (c) Given that the second derivative of the function F is

$$F''(x) = -\frac{6\left(4x^3 - 39x^2 + 108x - 117\right)}{\left(x^2-9\right)^3}$$

and that $F''(x) = 0$ only at $x \approx 6.12$, determine the intervals where the graph of F is concave up and concave down. Determine any inflection points of the graph of the function F.

[3] (d) Determine any vertical and horizontal asymptotes of the graph of the function F. Describe the behaviour of the graph of F at the vertical asymptotes using appropriate infinite limits.

[3] (e) Three graphs a shown below. Choose the one that best represents the graph of the function F. On the graph you choose indicate the coordinates of all of the points where there is a local maximum or minimum and any inflection points, and draw any vertical and horizontal asymptotes. Further, specify the x and y-intercepts of the graph.



(A) (B) (C)

19. Consider the equation

$$y \sin x - y \cos(x\,y) = 1$$

[3] (a) Use implicit differentiation to show that

$$\frac{dy}{dx} = \frac{y\left[\cos x + y \sin(xy)\right]}{\cos(xy) - xy \sin(xy) - \sin x}$$

[3] (b) Verify that the point $(\pi, 1)$ is on the graph of the equation above. Find an equation, in exact form (no decimals), of the tangent line to the graph of the equation above at this point. The branch of the graph of the equation above that goes through the point $(\pi, 1)$ is shown. Draw the tangent line that you found on this graph.