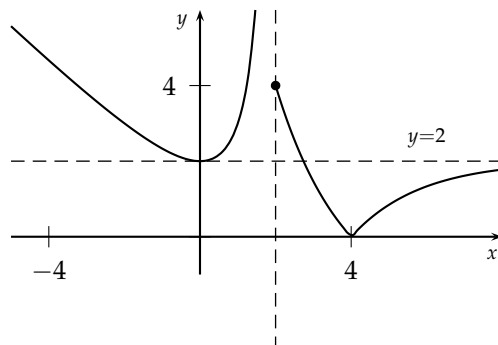


Example 1

The graph of a function f is shown. In parts (a) to (e) compute the limit if it exists. If the limit does not exist, determine if the limit is an infinite limit, and, if it is, state whether it is $+\infty$ or $-\infty$. Otherwise, explain why the limit does not exist.



- (a) $\lim_{x \rightarrow 4} f(x)$
 (b) $\lim_{x \rightarrow 2^-} f(x)$
 (c) $\lim_{x \rightarrow 2^+} f(x)$
 (d) $\lim_{x \rightarrow \infty} f(x)$
 (e) Give the point(s) where f is discontinuous. Classify any discontinuity as jump, infinite, or removable.

Example 2

Let

$$Q(x) = \begin{cases} x + 4 & \text{for } x < -1 \\ 5 & \text{for } x = -1 \\ x^2 + 5 & \text{for } -1 < x < 0 \\ 5 - x^2 & \text{for } 0 \leq x < 2 \\ x - 4 & \text{for } x \geq 2 \end{cases}$$

- (a) Sketch the graph of Q .
 (b) Find the value of each of the following limits, or explain why it does not exist.
 (i) $\lim_{x \rightarrow -1} Q(x)$ (ii) $\lim_{x \rightarrow 0^-} Q(x)$ (iii) $\lim_{x \rightarrow 0} Q(x)$ (iv) $\lim_{x \rightarrow 2^-} Q(x)$
 (c) Which of the following statements is true?
 (i) $\lim_{x \rightarrow 0} Q(x) = Q(0)$ (ii) $\lim_{x \rightarrow -1^+} Q(x) = Q(-1)$
 (iii) $\lim_{x \rightarrow -1^-} Q(x) = Q(-1)$ (iv) $\lim_{x \rightarrow 2^+} Q(x) = Q(2)$

Explain what these limits tell you about the continuity of Q at $x = -1$, $x = 0$, and $x = 2$.

Example 3

Find the value of each of the following limits, or explain why it does not exist.

- (a) $\lim_{x \rightarrow -1} \frac{x^2 - 2x - 3}{x^2 + 2x - 3}$
 (b) $\lim_{x \rightarrow -1} \frac{x^2 - 3x - 4}{x^2 + 3x + 2}$
 (c) $\lim_{x \rightarrow -1} \frac{\sqrt{x+5} - 2}{x^2 + 3x + 2}$

Example 4

Suppose that you are given a function k for which

$$\begin{aligned} \lim_{x \rightarrow -1^-} k(x) &= 1, & \lim_{x \rightarrow -1^+} k(x) &= 2, & \lim_{x \rightarrow 1^-} k(x) &= 3 \\ \lim_{x \rightarrow 1^+} k(x) &= 1, & \lim_{x \rightarrow 2^-} k(x) &= \infty, & \lim_{x \rightarrow 2^+} k(x) &= \infty \\ k(-1) &= 1, & k(1) &= 3 \end{aligned}$$

Sketch a possible graph of k .

Example 5

Evaluate each of the following limits and explain what the limit means in terms of a horizontal asymptote for the graph of the function.

(a) $\lim_{x \rightarrow \infty} \frac{3x^2 - 2x + 1}{4x^2 + 5x + 3}$ (b) $\lim_{x \rightarrow -\infty} \frac{x + 1}{\sqrt{x^2 + 1}}$

Example 6

Find the horizontal and vertical asymptotes for the function

$$f(x) = \frac{2x^2 + 3x - 2}{x^2 + x - 12}$$

Check the answer by graphing the curve and estimating the asymptotes.