
Example 1

Let $p(x) = \sqrt{5 - x^2}$.

(a) Find the linearization of p at $x = 2$.

(b) Use the linearization in part (a) above to approximate $p(1.9)$.

(c) Find the quadratic approximation

$$Q(x) = p(a) + p'(a)(x-a) + \tfrac{1}{2}p''(a)(x-a)^2$$

to the function $p(x)$ near $x = 2$ and use it to improve your estimate of $p(1.9)$.

Example 2

Use differentials (tangent line approximation) to estimate

(a) $\sqrt[4]{15.9}$ (b) $\sin 60.2^\circ$

Example 3

The frequency at which a stretched wire vibrates is given by the formula

$$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

where f is the frequency of vibration in hertz, L is length of the wire in meters, T is the tension in newtons, and ρ is the linear mass density of the wire in kg/m. The string on a guitar has a length of 55 cm and is held at a tension of 25 N. The linear mass density of the string is 40 g/m.

(a) Consider the frequency f as a function of the tension T, with the length and density held constant, find the differential df.

(b) Suppose that the tension in the wire is measured with a maximum possible error of 0.5 N. The other quantities are assumed to be known exactly. Use differentials to estimate the maximum possible error in the computed value of the frequency. Give both the absolute error and the relative error.

(c) Suppose that $y = \dfrac{u}{v}$ where both u and v are functions of x. Show that

$$dy = \frac{v\,du - u\,dv}{v^2}$$

(d) Suppose that the tension in the wire is measured with a maximum possible error of 0.5 N and the length is measured with a maximum possible error of 0.1 cm. The linear mass density is assumed to be known exactly. Use differentials, with the result from part (c) above, to estimate the maximum possible error in the computed value of the frequency. Give both the absolute error and the relative error.

Example 4

Martin borrows \$30000 to buy a new car at an interest rate of 0.4% per month with a term of 60 months. He wants to determine the effect of changes in interest rate on the total cost of borrowing money for the car. His accountant tells him that for an interest rate i per month with a term of n months, the total cost of borrowing money is

$$C = \frac{30000ni}{1-(1+i)^{-n}}$$

(a) Taking n to be a constant, calculate $\dfrac{dC}{di}$ for the formula above. You should not try to simplify this derivative too much.

(b) Find the value of $\dfrac{dC}{di}$ for the interest rate and term of Martin's loan. Remember that 0.4% = 0.004. Use differentials, or a linear approximation, to estimate how much the total cost of the loan will change if the interest rate increases from 0.4% to 0.5%.

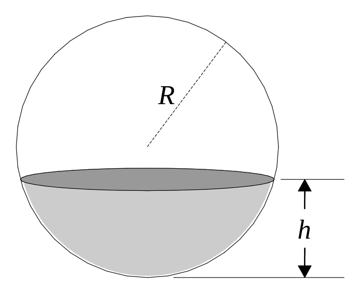
(c) Taking i to be constant, calculate $\dfrac{dC}{dn}$ for the formula above.

(d) Find the value of $\dfrac{dC}{dn}$ for the interest rate and term of Martin's loan. Use differentials, or a linear approximation, to estimate how much the total cost of the loan will change when the term of the loan increases from 60 to 62 months.

Example 5

A spherical segment is formed by cutting a sphere perpendicular to the axis. See the diagram. The volume of a spherical segment of depth h cut from a sphere of radius R is

$$V = \frac{\pi}{3}h^2\,(3R-h)$$



A spherical tank has a 40 cm radius. The depth of water in the tank is measured to 25 ± 0.1 cm. Estimate the error in the calculated volume of the water in the tank. Give both the absolute and relative errors.