Okanagan College

Math 122 (071) Winter 2010

Term Test One – Marksheet

Instructor: Clint Lee

Wednesday, February 3

Student Name: ______________________ Total Marks: ________ / 40

Problem	Marks	
1 Total		/2
2 Total		/2
3 Total		/2
4 Total		/2
5 Total		/2
6 Total		/2
7 (a)	/2	
7 (b)	/3	
7 Total		/5
8 (a)	/2	
8 (b)	/3	
8 Total		/6
9 (a)	/3	
9 (b)	/3	
9 (c)	/3	
9 Total		/9
10 Total		/4
11 (a)	/2	
11 (b)	/3	
11 Total		/5
Exam Total		/40

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Instructions. Do all parts of all 11 questions. Show all work and give explanations where required. You may receive part marks for a question if your work is correct even if the final answer is incorrect. However, if your answer is incorrect and no work or explanation is given, you will receive no marks. The number of points for each question is given in the left margin, total 40.

In problems 1 through 6 give a brief answer. You will be marked only on your answer, not on your work.

[2] 1. Find the most general antiderivative of $f(x) = e^{2x+1} + \cos(2x+1)$.

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[2] 2. Find the antiderivative of the function $g(x) = \sqrt{\frac{1}{2}x + 3}$ whose graph goes through the point $(2, 4)$.

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[2] 3. Set up the n subinterval right-endpoint Riemann sum for the definite integral $\int_0^2 x^3 \, dx$.

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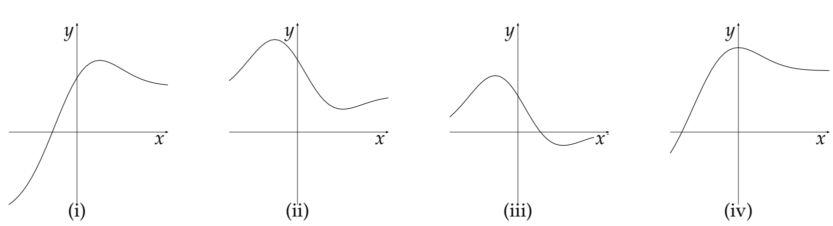
[2] 4. The graphs of four functions are shown. One is the graph of a function f and at least one other is the graph of an antiderivative of the function f. Identify the graph of the function f and the graph or graphs of an antiderivative. Explain your choices.



[2] 5. A function f is decreasing and concave up on the interval $[-2, 3]$. If you use R_{20}, L_{20}, T_{20}, M_{20}, and S_{20} to estimate the value of the definite integral $\int_{-2}^{3} f(x)\,dx$, which give an overestimate and which an underestimate. Explain your answer.

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[2] 6. Fill in the two blanks in the statement of the Fundamental Theorem of Calculus, Part 2 given below:

Suppose that the function f is continuous on the interval $[a, b]$ and that F is an antiderivative of f, so that

$$F'(x) = \_\_\_\_\_\_\_\_$$

Then

$$\int_a^b f(x)\,dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

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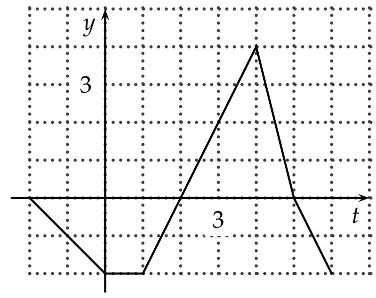
7. Water flows into my rain barrel during a downpour. Unfortunately water leaks out of the barrel at the same time. The table below shows the rate of inflow, $I(t)$, and the rate of outflow, $O(t)$, at five minute intervals over first thirty minutes of the downpour. At the beginning of the downpour the barrel held 25 litres of water.

t, min	0	5	10	15	20	25	30
$I(t)$, L/min	2.0	2.7	3.6	3.0	2.4	1.5	0.5
$O(t)$, L/min	0.5	0.8	1.0	1.5	1.9	2.2	2.5

[2] (a) Write an expression involving a definite integral in terms of the functions $I(t)$ and $O(t)$ that gives the total amount of water in the barrel at the end of the thirty minute period.

[3] (b) Use the Trapezoid Rule with $n = 3$ to estimate the value of the definite integral in part (a) above and give an estimate of the total amount of water in the barrel at the end of the thirty minute period.

8. Let $Q(x) = \int_{-2}^{x} h(t)\, \mathrm{d}t$ where $-2 \leq x \leq 6$ and the graph of h is as shown.



[2] (a) Find $Q(-2)$ and $Q(6)$.

[3] (b) Determine the intervals over which Q is increasing and decreasing and classify any critical points of Q as a local maximum, local minimum, or neither.

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9. Evaluate each integral. Give the **exact** value of any definite integral.

[3] (a) $\int t^2 \left(3 + 4t^3\right)^8 \, \mathrm{d}t$.

[3] (b) $\int_0^{\pi/4} \sec^2 \theta \sqrt{1 + \tan\theta}\, \mathrm{d}\theta$.

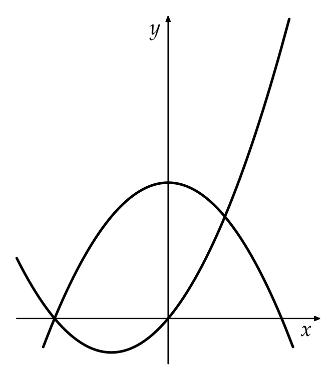
[3] (c) $\int_1^6 \dfrac{x+1}{\sqrt{x+3}}\, \mathrm{d}x$ using the substitution $z = \sqrt{x+3}$.

[4] 10. Use whichever of the sum formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \text{and} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

that are necessary to write the Riemann sum in Problem 3 without using summation notation, and evaluate an appropriate limit to find the value of the definite integral $\int_0^2 x^3 \, dx$.

[2] 11. (a) The graphs of the functions $y = x^2 + 3x$ and $y = 9 - x^2$ are shown. Label each of the graphs indicating which function is which and shade the portion of the diagram showing the region bounded by the graphs of these two functions over the interval $-3 \leq x \leq 3$.



[3] (b) Set up one or more definite integrals giving the area of the region bounded by the graphs of the functions $y = x^2 + 3x$ and $y = 9 - x^2$ over the interval $-3 \leq x \leq 3$. In setting up the integral(s) draw at least one vertical typical approximating rectangle on the graph above and give an expression for each approximating rectangle.