Okanagan College

Math 122 (071) Winter 2012

Term Test Two

Instructor: Jason Schaad

Wednesday, February 15

Student Name: ______________________ Total Marks: ________ / 38

Instructions. Do all parts of all 11 questions. Show all work and give explanations where required. You may receive part marks for a question if your work is correct even if the final answer is incorrect. However, if your answer is incorrect and no work or explanation is given, you will receive no marks. The number of points for each question is given in the left margin, total 38. A Formula Sheet is attached. You may use any of the formulas from this sheet. If you use an integral formula from the sheet, give the number of the formula that you used.

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[2] 1. Evaluate $\int \frac{x}{1+x^2}\,dx$.

[3] 2. Evaluate $\int_0^1 \sin\left(\frac{\pi}{2}t\right)\,dt$.

[2] 3. Evaluate $\int x\sin(x)dx$.

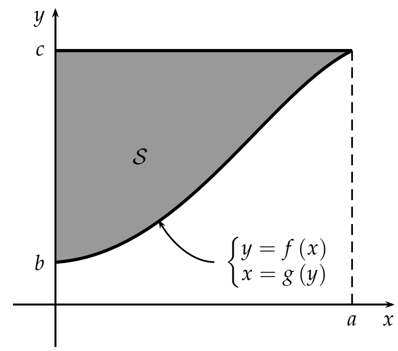
[2] 4. (a) Evaluate the indefinite integral $\int x \ln x \, \mathrm{d}x$.

[2] (b) Evaluate the indefinite integral $\int \frac{\ln x}{x} \, \mathrm{d}x$.

[2] 5. Evaluate $\int r^2 \sqrt{4+3r^3}\,dr$.

[3] 6. Evaluate $\int_0^7 \frac{x}{\sqrt[3]{x+1}}\,dx$.

[4] 7. Evaluate $\int x^2 e^x\,dx$.



8. Consider the region bounded by the curve $y = f(x) \Leftrightarrow x = g(y)$ and the y-axis, as shown.

[2] (a) Set up a definite integral for the area of the region $\mathcal{S}$ using vertical approximating rectangles.

[2] (b) Set up a definite integral for the area of the region $\mathcal{S}$ using horizontal approximating rectangles.

[4] 9. A metal rod of length π is centred at the origin. The temperature of the rod at position x is given by $T(x) = 20 + 10\cos(x)$ (in degrees Celsius), on the interval $[\frac{-\pi}{2}, \frac{\pi}{2}]$. Find the average temperature of the rod on this interval.

[5] 10. Use the method of slicing (i.e., disk or washer method) to find the volume of the solid obtained by rotating the region bound by the curves $y = 1/x$, $x = 1$, $x = 4$, $y = 0$ about the line $y = 1$. Sketch the region as well as a typical disk or washer.

[5] 11. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$, $y = 1$, $x = 0$ about the y-axis. Sketch the region and a typical shell.

A Short Table of Integrals

1. $\int f(g(x))g'(x)\,dx = \int f(u)\,du$ where $u = g(x)$
2. $\int u\,dv = vu - \int v\,du$
3. $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C \quad (n \neq -1)$
4. $\int \frac{du}{u} = \ln|u| + C$
5. $\int e^u\,du = e^u + C$
6. $\int a^u\,du = \frac{1}{\ln a}a^u + C$
7. $\int \sin u\,du = -\cos u + C$
8. $\int \cos u\,du = \sin u + C$
9. $\int \sec^2 u\,du = \tan u + C$
10. $\int \sec u \tan u\,du = \sec u + C$
11. $\int \csc^2 u\,du = -\cot u + C$
12. $\int \csc u \cot u\,du = -\csc u + C$
13. $\int \tan u\,du = \ln|\sec u| + C$
14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
15. $\int \cot u\,du = -\ln|\csc u| + C$
16. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
17. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\left(\frac{u}{a}\right) + C$
18. $\int \frac{du}{a^2 + u^2} = \left(\frac{1}{a}\right)\arctan\left(\frac{u}{a}\right) + C$
19. $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln\left|u + \sqrt{u^2 - a^2}\right| + C$
20. $\int \frac{du}{u^2 - a^2} = \left(\frac{1}{2a}\right)\ln\left|\frac{u-a}{u+a}\right| + C$
21. $\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du$
22. $\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du$
23. $\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\,du$
24. $\int \sec^n u\,du = \frac{1}{n-1}\tan u \sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$
25. $\int e^{au}\sin bu\,du = \frac{e^{au}}{a^2 + b^2}(a \sin bu - b \cos bu) + C$
26. $\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2 + b^2}(a \cos bu + b \sin bu) + C$

Short List of Identities

1. $\cos^2 x + \sin^2 x = 1$
2. $1 + \tan^2 x = \sec^2 x$
3. $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
4. $\sin 2x = 2 \sin x \cos x$
5. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
6. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

Series

The Geometric series is

$$\sum_{i=0}^{n-1} ar^i = a\left(\frac{1 - r^n}{1 - r}\right) = a\left(\frac{r^n - 1}{r - 1}\right)$$