Okanagan College

Math 122 (071) Winter 2012

Term Test Two

Instructor: Jason Schaad

Wednesday, February 15

Student Name: KEY

Total Marks: _____ / 38

Instructions. Do all parts of all 11 questions. Show all work and give explanations where required. You may receive part marks for a question if your work is correct even if the final answer is incorrect. However, if your answer is incorrect and no work or explanation is given, you will receive no marks. The number of points for each question is given in the left margin, total 38. A Formula Sheet is attached. You may use any of the formulas from this sheet. If you use an integral formula from the sheet, give the number of the formula that you used.

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[2] 1. Evaluate $\int \frac{x}{1+x^2}\,dx$.

$u = 1+x^2$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}du$

$$\int \frac{x}{1+x^2}dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln(1+x^2) + C = \boxed{\frac{\ln\sqrt{1+x^2}}{2} + C}$$

[3] 2. Evaluate $\int_0^1 \sin\left(\frac{\pi}{2}t\right)\,dt$.

$u = \frac{\pi}{2}t$

$du = \pi/2\,dt$

$\Rightarrow dt = \frac{2}{\pi}du$

$t = 0 \Rightarrow u = 0$

$t = 1 \Rightarrow u = \pi/2$

$$\int_0^1 \sin \pi/2\, t\,dt$$

$$= \frac{2}{\pi}\int_0^{\pi/2} \sin u\,du$$

$$= \frac{-2}{\pi}\cos u\Big|_0^{\pi/2} = \frac{-2}{\pi}\left[\cos(\pi/2) - \cos(0)\right]$$

$$= \boxed{2/\pi}$$

[2] 3. Evaluate $\int x\sin(x)dx$.

$u = x \quad dv = \sin x\,dx$

$du = dx \quad v = -\cos x$

$$\int x\sin(x)dx = x(-\cos x) - \int(-\cos x)dx = -x\cos x + \int \cos x\,dx$$

$$= \boxed{-x\cos x + \sin x + C}$$

[2] 4. (a) Evaluate the indefinite integral $\int x \ln x \, dx$.

Integration by parts

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{2}x^2$

then $\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \left(\frac{1}{2}x^2\right)\left(\frac{1}{x}dx\right)$

$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

$= \boxed{\frac{1}{4}x^2(2\ln x - 1) + C}$

[2] (b) Evaluate the indefinite integral $\int \frac{\ln x}{x} \, dx$.

substitution

$u = \ln x$

$du = \frac{1}{x}dx$

then $\int \frac{\ln x}{x} dx = \int u \, dx = \frac{1}{2}u^2 + C = \boxed{\frac{1}{2}(\ln x)^2 + C}$