Okanagan College

Math 122 (071) Winter 2012

Term Test Two

Instructor: Jason Schaad

Wednesday, February 15

Student Name: KEY

Total Marks: ______ / 38

Instructions. Do all parts of all 11 questions. Show all work and give explanations where required. You may receive part marks for a question if your work is correct even if the final answer is incorrect. However, if your answer is incorrect and no work or explanation is given, you will receive no marks. The number of points for each question is given in the left margin, total 38. A Formula Sheet is attached. You may use any of the formulas from this sheet. If you use an integral formula from the sheet, give the number of the formula that you used.

---

[2] 1. Evaluate $\int \frac{x}{1+x^2}\, dx$.

$u = 1+x^2$

$du = 2x\,dx$

$x\,dx = \frac{1}{2} du$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln(1+x^2) + C = \boxed{\ln \frac{\sqrt{1+x^2}}{2} + C.}$$

[3] 2. Evaluate $\int_0^1 \sin\left(\frac{\pi}{2}t\right)\, dt$.

$u = \frac{\pi}{2} t$

$du = \pi/2\, dt$

$\Rightarrow dt = \frac{2}{\pi} du$

$t = 0 \Rightarrow u = 0$

$t = 1 \Rightarrow u = \pi/2$

$$\int_0^1 \sin \tfrac{\pi}{2} t\, dt$$

$$= \frac{2}{\pi}\int_0^{\pi/2} \sin u\, du$$

$$= \frac{-2}{\pi}\cos u \Big|_0^{\pi/2} = \frac{-2}{\pi}\left[\cos(\pi/2) - \cos(0)\right]$$

$$\boxed{= 2/\pi}$$

[2] 3. Evaluate $\int x\sin(x)dx$.

$u = x \quad dv = \sin x\, dx$

$du = dx \quad v = -\cos x$

$$\int x\sin(x)dx = x(-\cos x) - \int(-\cos x)dx = -x\cos x + \int \cos x\, dx$$

$$= \boxed{-x\cos x + \sin x + C}$$

[2] 4. (a) Evaluate the indefinite integral $\int x \ln x \, dx$.

Integration by parts

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{2}x^2$

then $\int x \ln x\,dx = \frac{1}{2}x^2 \ln x - \int (\frac{1}{2}x^2)(\frac{1}{x}dx)$

$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

$= \boxed{\frac{1}{4}x^2(2\ln x - 1) + C}$

[2] (b) Evaluate the indefinite integral $\int \frac{\ln x}{x} \, dx$.

substitution

$u = \ln x$

$du = \frac{1}{x}dx$

then

$\int \frac{\ln x}{x}dx = \int u\,dx = \frac{1}{2}u^2 + C = \boxed{\frac{1}{2}(\ln x)^2 + C}$

[2] 5. Evaluate $\int r^2\sqrt{4+3r^3}\,dr$.

$u = 4+3r^3$

$du = 9r^2dr$

$\Rightarrow r^2dr = \frac{1}{9}du$

$\int r^2\sqrt{4+3r^3}\,dr = \int (4+3r^3)^{1/2}(r^2dr)$

$= \frac{1}{9}\int u^{1/2}du$

$= \frac{1}{9}\left(\frac{2}{3}\right)u^{3/2} + C = \frac{2}{27}(4+3r^3)^{3/2} + C$

[3] 6. Evaluate $\int_0^7 \frac{x}{\sqrt[3]{x+1}}\,dx$.

$u = \sqrt[3]{x+1}$

$x = u^3 - 1$

$\Rightarrow dx = 3u^2du$

$x = 0 \Rightarrow u = 1$

$x = 7 \Rightarrow u = 2$

$\int_0^7 \frac{x}{\sqrt[3]{x+1}}\,dx = \int_1^2 \frac{u^3-1}{u}(3u^2du)$

$= 3\int_1^2 (u^4 - u)\,du = 3\left(\frac{1}{5}u^5 - \frac{1}{2}u^2\right)\Big|_{u=1}^{u=2}$

$= 3\left[\left(\frac{32}{5} - 2\right) - \left(\frac{1}{5} - \frac{1}{2}\right)\right] = \boxed{\frac{141}{10}}$

[4] 7. Evaluate $\int x^2e^x\,dx$.

$u = x^2 \qquad dv = e^xdx$

$du = 2xdx \qquad v = e^x$

$* \quad \int x^2e^xdx = x^2e^x - 2\int xe^xdx$

$u = x \qquad dv = e^xdx$

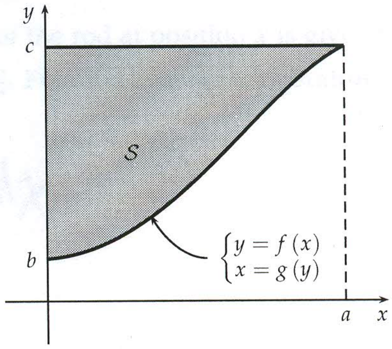
$du = dx \qquad v = e^x$

$\int xe^xdx = xe^x - \int e^xdx = xe^x - e^x + C$

plug into * gives

$\int x^2e^xdx = x^2e^x - 2(xe^x - e^x) = \boxed{x^2e^x - 2xe^x + 2e^x + C}$

8. Consider the region bounded by the curve $y = f(x) \Leftrightarrow x = g(y)$ and the y-axis, as shown.



[2] (a) Set up a definite integral for the area of the region S using vertical approximating rectangles.

The area of a vertical approximating rectangle is

$$\Delta A = [c - f(x)]\,dx$$

& to sweep out the whole region x goes from $x=0$ to $x=a$. So the area of the region S is

$$\boxed{A = \int_0^a [c - f(x)]\,dx}$$

[2] (b) Set up a definite integral for the area of the region S using horizontal approximating rectangles.

The area of a horizontal typical rectangle is

$$\Delta A = g(y)\Delta y$$

and to sweep out the whole region y goes from $x = b$ to $x = c$. So the area of the region S is

$$\boxed{A = \int_b^c g(y)\,dy}$$

[4] 9. A metal rod of length π is centred at the origin. The temperature of the rod at position x is given by $T(x) = 20 + 10\cos(x)$ (in degrees Celsius), on the interval $[\frac{-\pi}{2}, \frac{\pi}{2}]$. Find the average temperature of the rod on this interval.

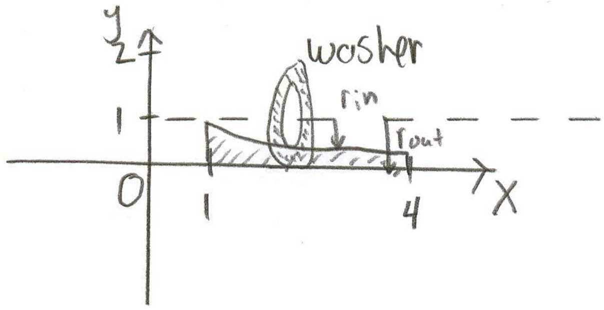
$$T_{ave} = \frac{1}{\pi/2 - (-\pi/2)} \int_{-\pi/2}^{\pi/2} (20 + 10\cos x)\,dx$$

$$= \frac{1}{\pi}\left[20x + 10\sin x\right]\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{\pi}\left[\left(20\tfrac{\pi}{2} + 10\sin(\tfrac{\pi}{2})\right) - \left(20(-\tfrac{\pi}{2}) + 10\sin(-\tfrac{\pi}{2})\right)\right]$$

$$= \boxed{20 + \frac{20}{\pi} \text{ degrees Celsius}}$$

[5] 10. Use the method of slicing (i.e., disk or washer method) to find the volume of the solid obtained by rotating the region bound by the curves $y = 1/x$, $x = 1$, $x = 4$, $y = 0$ about the line $y = 1$. Sketch the region as well as a typical disk or washer.



Slices are washers

$r_{out} = 1$

$r_{in} = 1 - \frac{1}{x}$

$$Vol = \pi \int_1^4 (r_{out}^2 - r_{in}^2)\,dx = \pi \int_1^4 1 - (1 - \tfrac{1}{x})^2\,dx$$

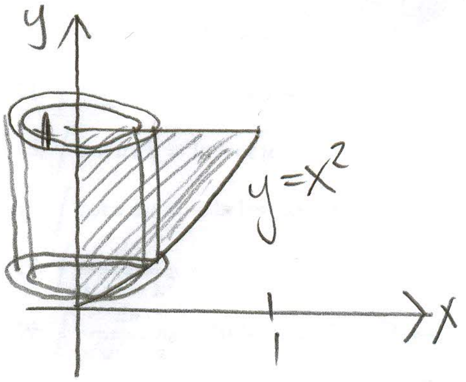
$$= \pi \int_1^4 \left[1 - (1 - \tfrac{2}{x} + \tfrac{1}{x^2})\right]dx = \pi \int_1^4 \left(\frac{2}{x} - \frac{1}{x^2}\right)dx$$

x^{-2}

$$= \pi\left(2\ln|x| + \frac{1}{x}\right)\Big|_1^4 = \pi\left[\left(2\ln 4 + \tfrac{1}{4}\right) - (2\ln 1 + 1)\right]$$

$$= \boxed{\pi\left(2\ln 4 - \frac{3}{4}\right)}$$

[5] 11. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$, $y = 1$, $x = 0$ about the y-axis. Sketch the region and a typical shell.



Shell radius $= x$

Shell height $= 1 - x^2$

$$\text{Vol} = 2\pi \int_0^1 (\text{radius})(\text{height})\,dx$$

$$= 2\pi \int_0^1 x(1-x^2)\,dx = 2\pi \int_0^1 (x - x^3)\,dx$$

$$= 2\pi \left(\tfrac{1}{2}x^2 - \tfrac{1}{4}x^4\right)\Big|_0^1 = 2\pi\left(\tfrac{1}{2} - \tfrac{1}{4}\right) - 0 = \boxed{\pi/2}$$