Okanagan University College
Math 112(71 & 72), Fall 1999
Term Test One

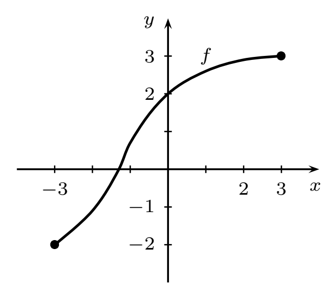
Instructor: Clint Lee

990928/29

Student Name: ______________________ Total Marks: ________ 40

Instructions. Do all parts of all 7 questions. Show all work and give explanations where required. You may receive part marks for a question if your work is correct even if the final answer is incorrect. However, if your answer is incorrect and no work or explanation is given, you will receive no marks. The number of points for each question is given in the left margin, total 40.

1. The graph of the function f is shown.



[2] (a) Give the domain and range of f.
Domain: $[-3, 3]$ Range: $[-2, 3]$

[1] (b) Explain why f is one-to-one.
f is one-to-one since it satisfies the horizontal line test, i.e., there is no pair of x values, x_1 and x_2 for which $f(x_1) = f(x_2)$.

[2] (c) Give the domain and range of f^{-1}.
Domain f^{-1} = Range $f = [-2, 3]$ and Range f^{-1} = Domain $f = [-3, 3]$

[1] (d) Estimate $f(2)$.
$f(2) \approx 2.7$

[1] (e) Estimate $f^{-1}(2)$.
$f^{-1}(2) \approx 0$ since $f(0) \approx 2$

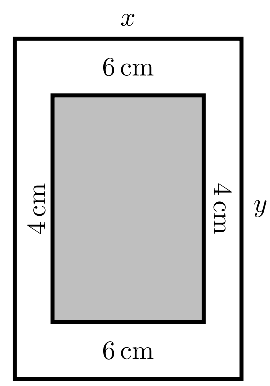
2. Let $f(x) = x + \dfrac{1}{x}$. Find and simplify

[2] (a) $f(1+h) = 1 + h + \dfrac{1}{1+h} = \dfrac{2 + 2h + h^2}{1+h}$

[2] (b) $f(1+h) - f(1) = \dfrac{2 + 2h + h^2}{1+h} - 2 = \dfrac{2 + 2h + h^2 - 2 - 2h}{1+h} = \dfrac{h^2}{1+h}$

[2] (c) $\dfrac{f(1+h) - f(1)}{h}$ for $h \neq 0$.

$$\frac{f(1+h) - f(1)}{h} = \frac{1}{h}\frac{h^2}{1+h} = \frac{h}{1+h}$$



[4] 3. The top and bottom margins of a poster are each 6 cm and the. side margins are each 4 cm. The area of the printed material on the poster is fixed at 384 cm^2. Express the total area of the poster as a function of its overall width x.
The area of the printed material of the poster is $(y-12)(x-8)$. Thus

$$\begin{aligned}(y-12)(x-8) = 384 &\Rightarrow y - 12 = \frac{384}{x-8}\\ &\Rightarrow y = 12 + \frac{384}{x-8}\end{aligned}$$

So that the total area of the poster is

$$\begin{aligned}A = xy &= x\left(12 + \frac{384}{x-8}\right)\\ &= 12x + \frac{384x}{x-8} = \frac{12x^2 - 96x + 384x}{x-8} = \frac{12x^2 + 288x}{x-8}\\ &= \frac{12x(x+24)}{x-8}\end{aligned}$$

4. The manager of a tree planting operation finds that when she has 10 planters 1000 trees get planted in a day, and when she has 22 planters 2500 trees get planted.

[3] (a) Assuming that the relationship between the number of tree planters, x, and the number of trees planted per day, y, is linear, find y as a function of x.
The slope is $m = \dfrac{2500 - 1000}{22 - 10} = 125$. Thus the equation is

$$y - 1000 = 125(x - 10) \Rightarrow y = 125x - 250$$

[2] (b) What is the slope of the graph of the linear function in part (a)? What does this slope represent? Give its units.
The slope is 125 trees per planter and it represents the number of trees that will be planted by one additional planter.

[2] (c) What is the x-intercept of the graph of the function in part (a)? Why isn't it zero?
The x-intercept of the graph is found by setting $y = 0$. This gives

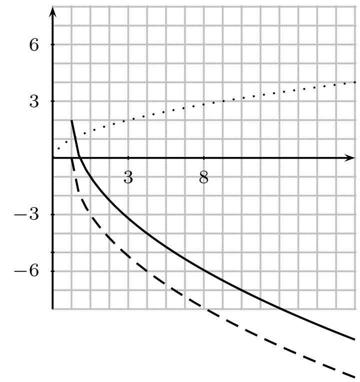
$$125x - 250 = 0 \Rightarrow x = 2$$

This value is not zero since if you only have two tree planters you can't get any trees planted, there is too much other work to get done.

[4] 5. The graph os $y = \sqrt{x}$ is shown. Use appropriate transformations to sketch the graph of

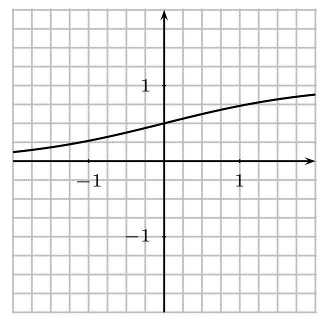
$$y = 2 - 3\sqrt{x-1}.$$

Describe each transformation and be sure to specify the order in which they must be applied.
The $x - 1$ shifts the graph right one unit, the $-$ in front of the root reflects the graph across the x-axis, multiplying the root by 3 stretches the graph vertically by a factor of three. The dashed graph results. Then adding 2 shifts the graph up by two units and the solid graph results.



6. Let $f(x) = \dfrac{e^x}{e^x + 1}$.

[3] (a) Sketch the graph of f and give its domain and range.
Domain: all real numbers
Range: $(-1, 1)$



[3] (b) Find a formula for $f^{-1}(x)$ and give its domain and range.

$$\begin{aligned} y = \frac{e^x}{e^x+1} &\Rightarrow ye^x + y = e^x \\ &\Rightarrow e^x(1-y) = y \\ &\Rightarrow e^x = \frac{y}{1-y} \\ &\Rightarrow x = \ln\left(\frac{y}{1-y}\right) \\ &\Rightarrow f^{-1}(x) = \ln\left(\frac{x}{1-x}\right) \end{aligned}$$

The domain of f^{-1} is the range of f which is $(-1, 1)$, and the range of f^{-1} is the domain of f which is all real numbers.

7. A bacterial population increases by 20% every hour. There are initially 1000 organisms.

[3] (a) Express the number of organisms in the form $P = P_0 e^{kt}$ where t is the time in hours since the population started to grow.

$$1.2P_0 = P_0 e^{k(1)} \Rightarrow e^k = 1.2 \Rightarrow k = \ln 1.2 = 0.1823$$

Thus

$$P = 1000e^{0.1823t}$$

Part marks were given for $P = 1000e^{0.2t}$ which close but not quite correct.

[1] (b) Find the population after ten hours.
When $t = 10$

$$P = 1000e^{0.1823 \cdot 10} = 6191.7 \approx 6192$$

For $P = 1000e^{0.2t}$ the result is 7389.

[2] (c) Find the time it takes for the population to double.
Find t when $P = 2P_0 = 1000$. This gives

$$\begin{aligned} 2P_0 = P_0 e^{0.8123t} &\Rightarrow e^{0.8123t} = 2 \\ &\Rightarrow 0.1823t = \ln 2 \\ &\Rightarrow t = \frac{\ln 2}{0.1823} = \frac{\ln 2}{\ln 1.2} = 3.8\,\text{hours} \end{aligned}$$

For $P = 1000e^{0.2t}$ the result is 3.47 hours.